

Quantum BRST operators in extended BRST anti BRST formalism

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Abstract

The quantum BRST anti BRST operators are explicitly derived and the consequences related to correlation functions are investigated. The connection with the standard formalism and the loopwise expansions for quantum operators and anomalies in $\text{Sp}(2)$ approach are analyzed.

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1 Introduction

The most powerful method for covariantly quantizing gauge theories is based on the antibracket-antifield formalism [1]-[3]. This technique has been extended by Batalin, Lavrov, Tyutin, in order to include the anti-BRST symmetry [4]-[7].

The geometrical interpretation given by Witten in terms of multivectors on the supermanifold of the fields was also generalized to the $\text{Sp}(2)$ -symmetric

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formalism, using a description in an overcomplete basis [8] and introducing further variables ($\bar{\phi}^A$) that kill the redundancy in cohomology. This allowed one to prove the equivalence of the two approaches and to express the quantum master equation in extended BRST anti BRST formalism as the condition that $\exp\left(\frac{i}{\hbar}W\right)$ is "divergencefree", just as in the standard case.

Moreover, the fact that the path-integral does not depend on the choice of the gauge-fixing function was written in a more elegant way in terms of the operator U_K and integration theory [8]:

$$\int [D\phi] A = \int [D\phi] (U_K A)_0 \quad (1)$$

$$U_{K'} A = U_K A + \bar{\Delta} C \quad (2)$$

for some C and if $\bar{\Delta} A = 0$, while:

$$U_K A = A + \bar{\Delta} J A \quad (3)$$

The contact with the well-known formulation in [4]-[5] is made by the choice:

$$U_K = \exp [K, \bar{\Delta}] \quad (4)$$

and:

$$\frac{i}{\hbar} K = \frac{\delta F}{\delta \phi^A} \frac{\delta}{\delta \phi_{B1}^*} - \frac{\delta F}{\delta \phi^A} \frac{\delta}{\delta \phi_{B2}^*} \quad (5)$$

for a gauge-fixing boson F .

However, the quantum aspects of the $\text{Sp}(2)$ theory were less addressed. The operators which implement the BRST-anti-BRST symmetry at quantum level were determined only in [10], using the "collective fields" as a basic tool, but a more direct computation is not available. The well-defined correlation functions and observables, the regularization and renormalization programs were not developed in this context.

The aim of this paper is to use the technique mentioned above in order to derive the expression of the quantum (anti) BRST operators in extended BRST formalism (section 2), to explore the consequences concerning the correlation functions (section 3) and to give an interpretation to the consistency conditions obtained for the $\text{Sp}(2)$ anomalies (section 4) showing the equivalence with certain previous results.

2 Deriving quantum (anti) BRST operators

The main consequence of the relations (1)-(3) is that the path-integral:

$$Z = \int [D\phi] U_K \exp\left(\frac{i}{\hbar} W\right) \quad (6)$$

does not depend on the choice of the gauge-fixing K , if the action W satisfies the quantum master equation which may be written as:

$$\overline{\Delta}\left(\exp\left(\frac{i}{\hbar} W\right)\right) = 0 \quad (7)$$

and has been shown to be equivalent with the following two equations in $\text{Sp}(2)$ formalism:

$$\overline{\Delta}^a \exp\left(\frac{i}{\hbar} W\right) = 0 \quad (8)$$

We use the well known denoting:

$$\overline{\Delta} = \overline{\Delta}^1 + \overline{\Delta}^2 \quad (9)$$

$$\overline{\Delta}^a = \Delta^a + \frac{i}{\hbar} V^a \quad (10)$$

The crucial requirement is now that a correlation functions:

$$I_K(B) \equiv \int [D\phi] U_K \exp\left(\frac{i}{\hbar} W\right) Y \quad (11)$$

should not depend on the gauge-fixing K . This in turn is true only if $\exp\left(\frac{i}{\hbar} W\right) Y$ is "divergencefree", which means the following condition:

$$\overline{\Delta}\left(\exp\left(\frac{i}{\hbar} W\right) Y\right) = 0 \quad (12)$$

must be fulfilled.

One can easily calculate the expression in (12) and find:

$$\begin{aligned} \Delta^a \left(\exp\left(\frac{i}{\hbar} W\right) Y \right) = \\ (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_{Aa}^*} \left(e^{\frac{i}{\hbar} W} Y \right) = \end{aligned}$$

$$\begin{aligned}
& e^{\frac{i}{\hbar}W} (-1)^{\varepsilon_A} \left(\left(\frac{i}{\hbar} \right)^2 \frac{\delta_l W}{\delta \phi^A} \frac{\delta W}{\delta \phi_{Aa}^*} Y + \frac{i}{\hbar} \frac{\delta_l W}{\delta \phi^A} \frac{\delta Y}{\delta \phi_{Aa}^*} + \right. \\
& \left. \frac{i}{\hbar} \frac{\delta^2 W}{\delta \phi^A \delta \phi_{Aa}^*} Y + \frac{i}{\hbar} \frac{\delta_l W}{\delta \phi_{Aa}^*} \frac{\delta Y}{\delta \phi^A} + \frac{\delta^2 Y}{\delta \phi^A \delta \phi_{Aa}^*} \right) = \\
& e^{\frac{i}{\hbar}W} \left[\frac{i}{\hbar} \left[\Delta^a W + \frac{i}{2\hbar} (W, W)^a \right] Y + \right. \\
& \left. \left[\Delta^a Y + \frac{i}{\hbar} (Y, W)^a \right] \right] \tag{13}
\end{aligned}$$

which gives:

$$\begin{aligned}
\overline{\Delta}^a (e^{\frac{i}{\hbar}W} Y) &= \left(\Delta^a + \frac{i}{\hbar} V^a \right) (e^{\frac{i}{\hbar}W} Y) = \\
e^{\frac{i}{\hbar}W} &\left[\frac{i}{\hbar} \left[\overline{\Delta}^a W + \frac{i}{2\hbar} (W, W)^a \right] Y + \right. \\
&\left. \left[\overline{\Delta}^a Y + \frac{i}{\hbar} (Y, W)^a \right] \right] \tag{14}
\end{aligned}$$

If the quantum master equations are true ($a = 1, 2$), it remains to require:

$$\overline{\sigma}^a Y = 0 \tag{15}$$

and thus:

$$\overline{\sigma} Y = (\overline{\sigma}^1 + \overline{\sigma}^2) Y = 0 \tag{16}$$

where:

$$\overline{\sigma}^a Y \equiv (Y, W)^a - i\hbar \overline{\Delta}^a Y \tag{17}$$

We thus obtained the explicit expressions of the operators $\overline{\sigma}^a$ which obviously tend to the classical BRST anti BRST operators:

$$s^a = (, S)^a + V^a \tag{18}$$

when the limit $\hbar \rightarrow 0$ is considered.

3 Correlation functions

We may now investigate the significance of the condition (16).

A first remark is that if we use a loopwise expansion for:

$$Y = \sum_{p=0}^{\infty} \hbar^p Y_p \quad (19)$$

and

$$W = S + \sum_{p=1}^{\infty} \hbar^p M_p \quad (20)$$

then the following set of relations is encoded in (16):

$$(Y_0, S)^a + V^a Y_0 \equiv s^a Y_0 = 0 \quad (21)$$

$$s^a Y_1 = i\Delta^a Y_0 - (Y_0, M_1) \quad (22)$$

$$s^a Y_p = i\Delta^a Y_{p-1} - \sum_{q=0}^{p-1} (Y_q, M_{p-q}), \quad p \geq 2 \quad (23)$$

One can see that the "classical" part of the quantum operator Y is a classical BRST-anti-BRST invariant.

A second remark concerns the possibility of finding "trivial" operators which would generate zero correlation functions.

Let us assume that $\bar{\sigma}Y = 0$ and:

$$Y = \bar{\sigma}X \quad (24)$$

Then the corresponding correlation function is:

$$\begin{aligned} I_K(Y) &= \int [D\phi] U_K \exp\left(\frac{i}{\hbar}W\right) Y = \\ &= \int [D\phi] \left(\exp\left(\frac{i}{\hbar}W\right) \bar{\sigma}X \right) + \int [D\phi] \bar{\Delta}J \left(\exp\left(\frac{i}{\hbar}W\right) \bar{\sigma}X \right) \end{aligned} \quad (25)$$

according to (3). The second term in (25) is zero due to the property

$$\int [D\phi] \bar{\Delta}A = 0 \quad (26)$$

(see [8]). It remains:

$$\begin{aligned} \int [D\phi] \left(\exp\left(\frac{i}{\hbar}W\right) \bar{\sigma}X \right) &= \int [D\phi] \bar{\Delta} \left(\exp\left(\frac{i}{\hbar}W\right) X \right) - \\ &= \int [D\phi] \exp\left(\frac{i}{\hbar}W\right) \left\{ \frac{i}{\hbar} \left[\bar{\Delta}W + \frac{i}{2\hbar} (W, W) \right] Y \right\} \end{aligned} \quad (27)$$

which is zero due the same (26) and quantum master equation.

We are led to the conclusion that in order to obtain good $I_K(Y)$ we must use operators which satisfy $\bar{\sigma}Y = 0$ but are not $\bar{\sigma}$ -exact (there is no X such that $Y = \bar{\sigma}X$).

On the other hand, the result:

$$\int [D\phi] U_K \exp\left(\frac{i}{\hbar}W\right) \bar{\sigma}X = 0 \quad (28)$$

for various functionals X produces identities among correlation functions. They embody the Ward identities associated to the extended BRST invariance of the theory and with a particular choice of X one can obtain, up to normalization factors, the associated Schwinger-Dyson equations.

4 Consistency conditions for $\text{Sp}(2)$ anomalies and discussions

The violation of the quantum master equation can be expressed as:

$$\mathcal{A}^a = \bar{\Delta}^a W + \frac{i}{2\hbar} (W, W)^a \quad (29)$$

which is then found to respect the "consistency condition" encoded in:

$$\bar{\sigma}^{\{a} \mathcal{A}^{b\}} = 0 \quad (30)$$

and to be equivalent with the following set of equations:

$$\left(\mathcal{A}_1^{\{a}, S\}^{b\}}\right) = iV^{\{a} (M_1, S)^{b\}} \quad (31)$$

$$\begin{aligned} \left(\mathcal{A}_p^{\{a}, S\}^{b\}}\right) &= iV^{\{a} (M_p, S)^{b\}} - \sum_{n=1}^{p-1} \left(\mathcal{A}_{p-n}^{\{a}, M_n\}^{b\}}\right) + i\Delta^{\{a} \mathcal{A}_{p-1}^{b\}} + \\ &iV^{\{a} \sum_{n=1}^{p-1} (M_{p-n}, M_n)^{b\}} \quad , \quad p \geq 2 \end{aligned} \quad (32)$$

if we expanded:

$$\mathcal{A}^a = \sum_{p=0}^{\infty} \hbar^p \mathcal{A}_p^a \quad (33)$$

and the definition (29) was explicitly developed as:

$$\mathcal{A}_0^a = \frac{1}{2} (S, S)^a + V^a S = 0 \quad (34)$$

$$\mathcal{A}_1^a = \Delta^a S + i (M_1, S)^a + i V^a M_1 \quad (35)$$

$$\mathcal{A}_p^a = \Delta^a M_{p-1} + i (M_p, S)^a + i V^a M_p + i \sum_{q=1}^{p-1} (M_q, M_{p-q})^a \quad (36)$$

The property (30) can be straightforwardly proven by using the definition (29) and the operator algebra for Δ^a, V^a given in [4]-[6].

A direct consequence is that:

$$s^{\{a} \mathcal{A}_1^{b\}} = 0 \quad (37)$$

$$s^{\{a} \mathcal{A}_p^{b\}} = - \sum_{n=1}^{p-1} \left(\mathcal{A}_{p-n}^{\{a}, M_n \}^{b\}} \right) + i \Delta^{\{a} \mathcal{A}_{p-1}^{b\}}, p \geq 2 \quad (38)$$

and one may easily see that (37), the one loop order condition, is exactly the one which has been previously deduced in a completely different way in [11]-[13], where it was written as: $sa^{-1} + \bar{s}a^1 = 0$.

We analyzed the quantization of the $\text{Sp}(2)$ - symmetric Lagrangian formalism, underlying the derivation of the correct quantum operators in this context, the relation with the standard case and the implications for the correlation functions and anomalous terms. We should also stress that the expressions involved in (8) or in (29) are not well-defined unless an appropriate regularization scheme is used and this is a topic to be covered in the near future.

References

- [1] B. de Wit and J. W. van Holten, Phys. Lett. B79 (1978)389.
- [2] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B102 (1981)27.

- [3] I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D28 (1983)2567.
- [4] I. A. Batalin, P. Lavrov and I. Tyutin, J. Math. Phys. 31(1990)1487.
- [5] I. A. Batalin, P. Lavrov and I. Tyutin, J. Math. Phys. 32(1990)532.
- [6] I. A. Batalin, P. Lavrov and I. Tyutin, J. Math. Phys. 32(1990)2513.
- [7] M. Henneaux, Nucl. Phys.B (Proc. Suppl.) 18A (1990)47.
- [8] M. Henneaux, hep-th/9205018.
- [10] P. H. Damgaard, F. De Jonghe and R. Sollacher, hep-th/9505040.
- [11] Brandt F., Dragon N., Kreuzer M., DESY 89-089, ITP-UH 5/83.
- [12] Brandt F., Dragon N., Kreuzer M., DESY 89-076, ITP-UH 2/89.
- [13] Brandt F., Dragon N., Kreuzer M., DESY 89-088, ITP-UH 6/89.
- [14] J. Paris and W. Troost, hep-th/9607215.
- [15] J. Paris, Nucl. Phys., B450 (1995) 357.
- [16] E. Witten, Mod. Phys. Lett. A5 (1990)487.
- [17] W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Nucl. Phys. B333(1990)727.